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where g=gravity=32, l=length to center of oscillation$=\frac{2}{3}a$, h=height of zero velocity$=2l$ without a sensible error, t=time, θ=angle counted from the down point. Reducing, we have,

$$t=\tfrac{1}{2}\sqrt{\left(\frac{l}{g}\right)}\int_{\frac{1}{2}\pi}^{\pi-\phi}\frac{d\theta}{\cos\frac{1}{2}\theta}=\sqrt{\left(\frac{l}{g}\right)}\Big[\log\tan(\tfrac{1}{4}\pi+\tfrac{1}{4}\theta)\Big]_{\frac{1}{2}\pi}^{\pi-\phi}$$

$$=\sqrt{\left(\frac{l}{g}\right)}\log\frac{\cot\frac{1}{4}\phi}{\tan\frac{3}{8}\pi}=\sqrt{\left(\frac{l}{g}\right)}(8.647491).$$

But $l=\frac{2}{3}$ of 100. $\therefore t=\sqrt{(\frac{100}{48})}(8.647491)=12.481$ seconds.

AVERAGE AND PROBABILITY.

139. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Four points are taken at random on the surface of a given sphere; find the average volume of the tetrahedron formed by the planes passing through the points taken three and three.

No solution has been received.

140. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Obtain the average area of a triangle formed by a tangent to the four-cusped hypocycloid and the co-ordinate axes.

Solution by J. E. SANDERS, Hackney, Ohio.

The curve's equation is $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ The length of the perpendicular from the origin on the tangent is

$$\frac{xdx-ydy}{\sqrt{(dx^2+dy^2)}}=\frac{x^{\frac{2}{3}}+y^{\frac{2}{3}}}{\sqrt{(y^{-\frac{2}{3}}+x^{-\frac{2}{3}})}}=\sqrt[3]{(axy)}.$$

The equation to the tangent is $x'/x^{\frac{1}{3}}+y'/y^{\frac{1}{3}}=a^{\frac{2}{3}}$, and its length between the axis$=a^{\frac{2}{3}}\sqrt{(x^{\frac{2}{3}}+y^{\frac{2}{3}})}=a$.

$\therefore$The area of the triangle is $A=\frac{1}{2}a\sqrt[3]{(axy)}=\frac{1}{2}a^2\sin\theta\cos\theta$, if $x=a\cos^3\theta$, and $y=a\sin^3\theta$.

Then, if the tangent is through any point on the curve, the average area is

$$\Delta=\frac{1}{s}\int Ads.$$

But $ds=a^{\frac{1}{3}}x^{-\frac{1}{3}}dx=3a\sin\theta\cos\theta\, d\theta$ and $s=\frac{3}{2}a$.

$\therefore\Delta=a^2\int_0^{\frac{1}{2}\pi}\sin^2\theta\cos^2\theta\, d\theta=\frac{1}{16}\pi a^2$, or, if the tangent is to be a random line, the average area is

$$\triangle_1=\frac{2}{\pi}\int_0^{\frac{1}{2}\pi}\tfrac{1}{2}a^2\sin\theta\cos\theta\,d\theta=\frac{a^2}{2\pi}.$$

As the problem is stated it seems that either of the above suppositions are allowable.

Also solved by the *PROPOSER*.

142. Proposed by ARTEMAS MARTIN, A. M., Ph. D., LL. D., Washington, D. C.

Two points are taken at random in the arc of a semi-circle, and a third point anywhere in its base. Find the probability that the triangle formed by them is acute.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

Let A, B be the random points on the arc of the semi-circle, MN its diameter or base, C its center. On AB as diameter describe a circle with center F. At A, B, perpendicular to AB, draw AD, BE meeting MN in D and E. From A draw AL parallel to MN meeting BE in L, and from F draw FG perpendicular to MN meeting MN in G. If the circle on AB does not intersect MN, the third point can be anywhere on DE; if the circle on AB intersects MN, then the third point can be anywhere on DE without this circle.

$\therefore$ If the circle on AB intersects MN in H, then the third point K can be anywhere on DH or EK. Let $CA=CB=r$, $\angle MCF=\theta$, $\angle ACF=\varphi$. Then $AB=2r\sin\varphi$, $AL=DE=2r\sin\varphi\operatorname{cosec}\theta=Q$, $FC=r\cos\varphi$, $FG=r\cos\varphi\sin\theta$. When $FG=FB$ we get $r\sin\varphi=r\cos\varphi\sin\theta$.

$\therefore \tan\varphi=\sin\theta$, $HG=\sqrt{(HF^2-FG^2)}=\sqrt{(AF^2-FG^2)}$.

$\therefore HG=r\sqrt{(\sin^2\varphi-\sin^2\theta\cos^2\varphi)}$.

$\therefore DH+EK=DE-2HG=2r[\sin\varphi\operatorname{cosec}\theta-\sqrt{(\sin^2\varphi-\sin^2\theta\cos^2\varphi)}]=Q'$.

Let $\varphi=\tan^{-1}(\sin\theta)=\varphi'$, p=required chance.

$$\therefore p=\frac{\left[\frac{1}{2r}\int_0^{\frac{1}{2}\pi}\int_0^{\phi'}Q\,d\theta\,d\varphi+\frac{1}{2r}\int_0^{\frac{1}{2}\pi}\int_{\phi'}^{\theta}Q'\,d\theta\,d\varphi\right]}{\int_0^{\frac{1}{2}\pi}\int_0^{\theta}d\theta\,d\varphi}$$

$$=\frac{8}{\pi^2}\int_0^{\frac{1}{2}\pi}\left[\int_0^{\phi'}\sin\varphi\operatorname{cosec}\theta\,d\varphi+\int_{\phi'}^{\theta}(\sin\varphi\operatorname{cosec}\theta\right.$$

$$\left.-\sqrt{(\sin^2\varphi-\sin^2\theta\cos^2\varphi)}d\,\varphi\right]d\theta=\frac{4}{\pi^2}+\frac{8}{\pi^2}\int_0^{\frac{1}{2}\pi}\int_{\phi}^{\theta}\frac{\tan^2\varphi\sec\varphi\,d\theta\,d\varphi}{\sqrt{(\tan^2\varphi-\sin^2\theta)}}.$$

The last term can be expanded and then integrated in series.

143. Proposed by L. C. WALKER, A.M.. Professor of Mathematics, Colorado School of Mines, Golden, Col.

The extremities of two equal lines drrwn from a fixed point in the circumference of a given circle is joined. Find the average area of the circle inscribed in the triangle formed.